Name:

Quiz 3 – 9/13/2023

Instructions. You have 25 minutes to complete this quiz. You may use your plebe-issue TI-36X Pro calculator. You may <u>not</u> use any other materials.

<u>Show all your work.</u> To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
2c	1	
Total		/ 70

Problem 1. You are a consultant for a political pollster in Simplexville. Each year, the citizens of Simplexville vote for one of three parties: (1) the Optimal Party, (2) the Unbounded Party, or (3) the Infeasible Party.

Based on historical data, you have determined that voting behavior in Simplexville can be modeled as a Markov chain with states $\mathcal{M} = \{1, 2, 3\}$ (1 = Optimal, 2 = Unbounded, 3 = Infeasible), and with each time step corresponding to one year. The one-step transition matrix is

$$\mathbf{P} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

For example, of those that voted for the Unbounded Party in this year's election, 10% will vote Optimal next year, 80% will vote Unbounded, and 10% will vote Infeasible.

Suppose in this year's election, 45% voted Optimal, 50% voted Unbounded, and 5% voted Infeasible.

a. Note that the diagonal entries of $\mathbf{P}$ are larger than the off-diagonal entries. What does that mean in this setting?

Here is the one-step transition matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.80 & 0.10 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

b. Suppose this year corresponds to time step $n = 0$. What is the probability that a citizen votes for the Unbounded Party 4 years from now ($n = 4$), given that the citizen voted for the Unbounded Party this year?

c. Again, suppose this year corresponds to time step $n = 0$. What is the probability that a randomly selected citizen votes for the Optimal Party 4 years from now ($n = 4$)?

d. What is the probability that a citizen votes for the Unbounded Party this year, votes either Unbounded or Infeasible for the next 3 years, and then votes for the Optimal Party 4 years from now?

Problem 2. Consider a Markov chain with state space $\mathcal{M} = \{1, 2, 3, 4, 5\}$ and transition probabilities defined by the matrix and diagram below:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.5 & 0.1 & 0.1 & 0.2 \\ 0 & 0.8 & 0 & 0 & 0.2 \\ 0 & 0 & 0.3 & 0 & 0.7 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0.5 \end{bmatrix}$$
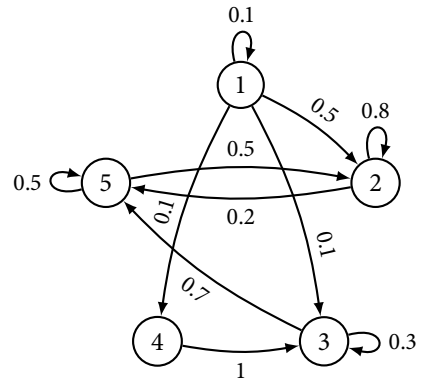
a. Is the set $\mathcal{R} = \{2, 5\}$ a recurrent class? Briefly explain.

b. Is state 5 transient or recurrent? Briefly explain.

c. Is state 1 transient or recurrent? Briefly explain.